Now, putting the origin at the centre, we have

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,,\quad x^2+y^2=r'^2\,,\quad \frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{p'^2}\,;$$

whence
$$a^2+b^2-r'^2=\frac{a^2b^2}{p'^2}\,,$$

$$\gamma'=p'\frac{dr'}{dp'}=\frac{a^2+b^2-r'^2}{r'}=\frac{r\,(2a-r)}{r'}\,,\ \text{by (6)},$$

$$\frac{\gamma'}{\gamma}=\frac{a}{r'}\,.$$

We have now

$$\gamma\varphi\,\frac{d}{dr}\left[\frac{F}{\varphi}\right]dr+\gamma'\varphi'\,\frac{d}{dr'}\left[\frac{F'}{\varphi'}\right]dr'=0$$

$$\gamma'=\frac{a\gamma}{r'}\,,\quad \varphi=\frac{\mu}{r^2}\,,\quad \varphi'=\mu' r'\,,\quad r'dr'=(r-a)dr\,.$$

Making the proper substitutions we obtain

$$\frac{1}{r^2(a-r)}\,\frac{d}{dr}\,(Fr^2)=\frac{a}{r'}\frac{d}{dr'}\left[\frac{F'}{r'}\right],$$

the same as (7). [*Joseph Bowden, Jr.*]

EXERCISES.

348

Prove that, if $0<\alpha<\beta$,

$$\int_\alpha^\beta \log\frac{\beta-x}{x-\alpha}\,\frac{dx}{x}=\frac{1}{2}\left[\log\frac{\beta}{\alpha}\right]\,.$$

[*Frank Morley.*]

349

Integrate the differential equation

$$dy=\text{arc sin }(x^2)\,dx.$$

[*Artemas Martin.*]

350

Let p_1, p_2, p_3 be the points of contact of parallel tangents to a cardioid. Let them be in positive order. Let $p_2 p_3 q_1$, $p_3 p_1 q_2$, $p_1 p_2 q_3$ be positive equilateral triangles. Prove that q_1, q_2, q_3 lie on a line parallel to the tangents.

[*Frank Morley.*]

351

The outer coatings of two condensers, A and B, are put to earth, and their inner coatings are connected together through a galvanometer, the resistance of which is g. The capacities of the condensers are C and c, respectively. Both are charged initially to the same potential difference, V_0, and then have charges of Q_0 and q_0, respectively. Show that if the inner coatings of A and B are put to earth simultaneously through wires of no self-induction, but of resistance R and r, respectively, the charge on A after t seconds will be

$$Q = \frac{Q_0}{2\varkappa}\,\varepsilon^{\frac{-t(\mu+\mu')}{2m}}\left\{\left[\varkappa+\mu+\mu'-\frac{2m}{CR}\right]\varepsilon^{\frac{\varkappa t}{2m}}+\left[\varkappa-\mu-\mu'+\frac{2m}{CR}\right]\varepsilon^{\frac{-\varkappa t}{2m}}\right\},$$

where $\lambda = rCR$, $\lambda' = rcR$,

$$\mu = cr\,(g+R)\,,\quad \mu' = CR\,(g+r)\,,$$

$$m = CcgRr\,,\ \text{ and }\ \varkappa^2 = 4\lambda\lambda' + (\mu-\mu')^2.$$

Show also that the whole quantity of electricity which passes through the galvanometer during the discharge, will be

$$M = \frac{Q_0\,\{C^2R^2(\mu\mu'-\lambda\lambda') + m^2 - mCR\,(\mu+\mu')\}}{C^2R^2\,(\mu\mu'-\lambda\lambda')} = \frac{Q_0(CR-cr)}{C\,(g+r+R)}\,.$$

It is to be noticed that $\mu\mu' - \lambda\lambda'$ can never be zero. If $CR = cr$, $M = 0$, as is the case in De Sauty's method of comparing the capacities of two condensers. In applying the expressions written above to numerical problems, one sometimes needs to know that one $\left\{\begin{array}{l}\text{microfarad}\\ \text{ohm}\\ \text{microcoulomb}\end{array}\right.$ is equivalent to $\left\{\begin{array}{l}10^{-15}\\ 10^{9}\\ 10^{-7}\end{array}\right.$ absolute electromagnetic units, and to $\left\{\begin{array}{l}9 \times 10^{5}\\ 9^{-1}\times 10^{-11}\\ 3 \times 10^{3}\end{array}\right.$ absolute electrostatic units of $\left\{\begin{array}{l}\text{capacity.}\\ \text{resistance.}\\ \text{quantity.}\end{array}\right.$

[*B. O. Peirce.*]